Data Assimilation - Precision Localization

miércoles, 23 de septiembre de 2020 02:19 p. m.

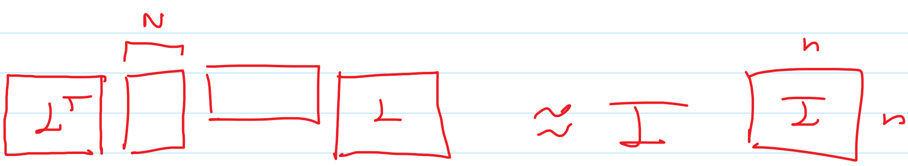
$$P^b = \hat{\Delta x}\,\hat{\Delta x}^T \qquad \varepsilon x^{b(e)} \sim \mathcal{N}(0, B)$$

$$\left.\text{SPAN}(\Delta X) = \text{SPAN}\Big(\underbrace{\tfrac{1}{\sqrt{N-1}}\Delta X}_{\hat{\Delta X}}\Big)\right\} \quad (*)$$

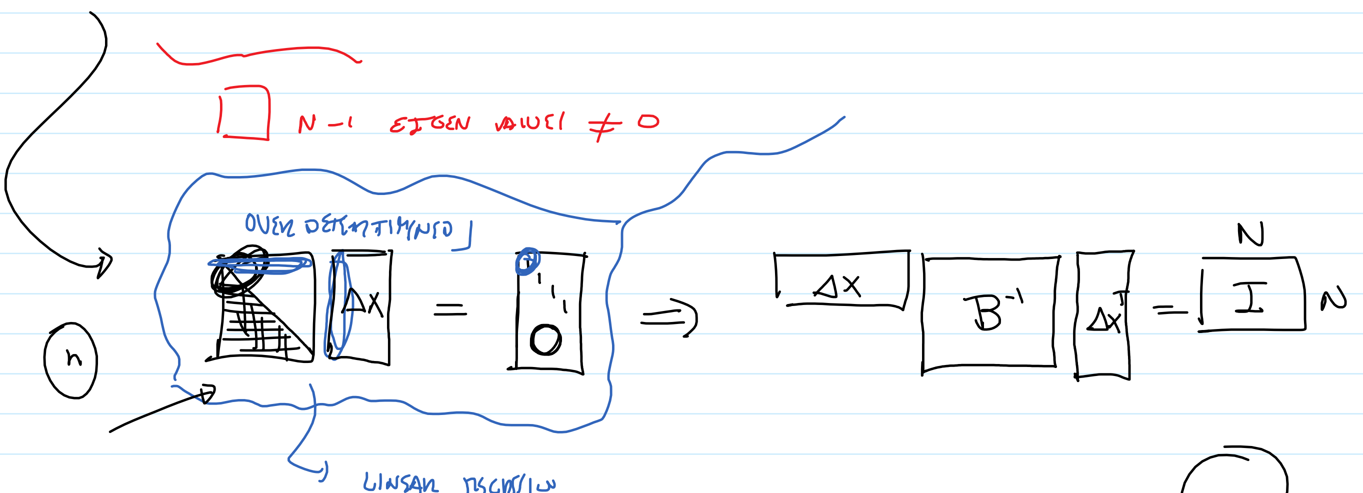
$$B^{-T/2} B B^{-1/2} = I \quad \mapsto \quad \underbrace{B^{-T/2} B^{T/2}}_{I}\,\underbrace{B^{1/2} B^{-1/2}}_{I} = I$$

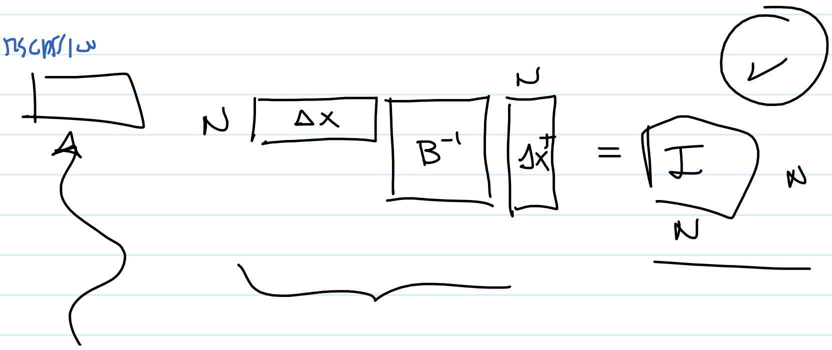
$$P^b = \frac{1}{N-1}\Delta X \Delta X^T = \frac{1}{\sqrt{N-1}}\Delta X \cdot \frac{1}{\sqrt{N-1}}\Delta X^T$$

$$L^T P^b L \approx I \qquad L^T \approx [P^b]^{1/2}$$

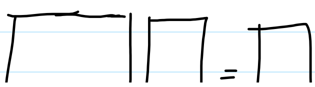


N-1 EIGEN VALUES $\neq 0$





$$Ax = b \qquad \|Ax - b\|$$



$$\underbrace{\boxed{\Delta x}\ \boxed{L^T}\ \boxed{L}\ \boxed{\Delta x}}\ = I$$

(1)

$$L = \begin{bmatrix} \diagdown & \\ & \diagdown \end{bmatrix}$$

1) LOWER TRIANGULAR MATRIX

$$x^{[i]} = \sum_{j \in P(i,r)} \beta_{ij} \cdot x^{[j]} + \varepsilon^{[i]}$$

$$\varepsilon^{[i]} \sim N(0, \sigma^2)$$

$$Q(i,r) \quad j \,/\, d(i,j)^2 \le r^2$$

$Q(i,r)$

$$P(i,r) \quad j \,|\, d(i,j)^2 \le r^2 \ \wedge\ j < i$$

$$\{L\}_{ij} = -\beta_{ij}$$

$$x^{[i]} = \sum_{j \in P(i,r)} \beta_{ij} \cdot x^{[j]} + \varepsilon^{[i]}$$

$$\varepsilon^{[i]} \sim N(0, \sigma^2)$$

$$\{D\}_{ii} = \mathrm{VAR}\Big(\underbrace{x^{[i]} - \sum_{j \in P(i,r)} \beta_{ij}\, x^{[j]}}_{\varepsilon^{[i]}}\Big)$$

$Q(6, r=1)$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

$$\boxed{B^{-1} = L^T \underbrace{L}}$$

L UNA RAÍZ CUADRADA B^{-1}

$$A = A^{1/2} A^{1/2} \rightarrow A = \overbrace{A^{1/2} Z^T}^{W^T} \overbrace{Z A^{1/2}}^{W} = \underbrace{W^T W}_{\# \boxed{\alpha}}$$

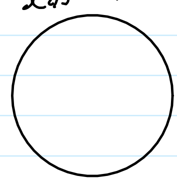
$$\underbrace{Z^T Z}_{I}$$

r - 1

r = 1

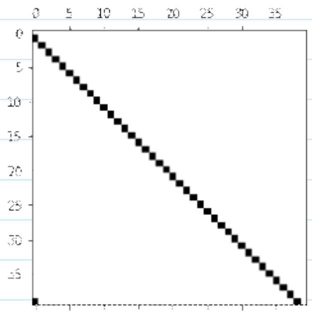
← 1 2 - 40   r = 1   → 1 40 →

1 LORENZ 96

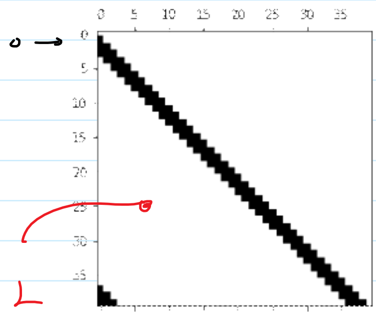


PREDECESSORES

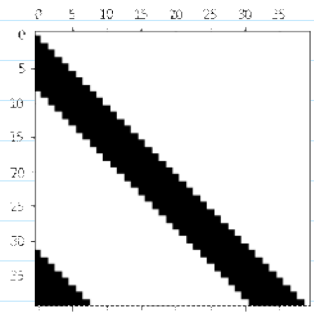
r = 1



r = 3



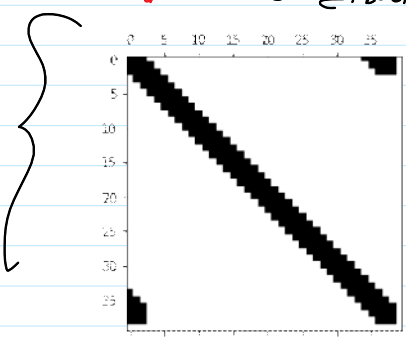
r = 5



STRUCT $B^{-1} = L^T \underbrace{D^{-1}}_{\text{DIAGONAL}} L$

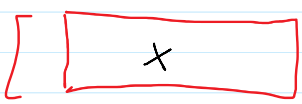
I

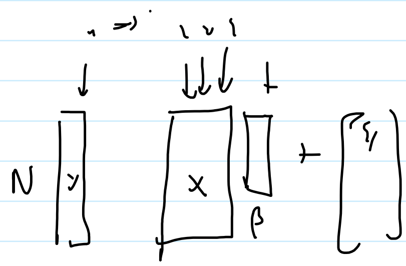
ESTRUCTURA PARA r = 3

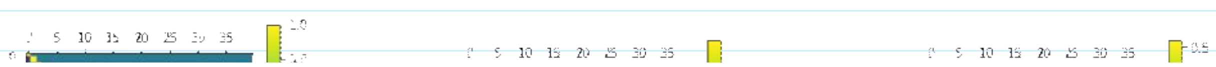


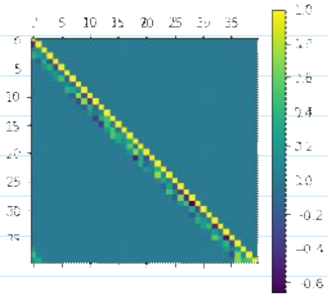
N

IND

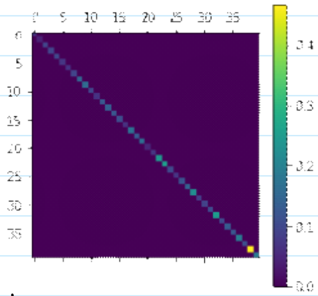




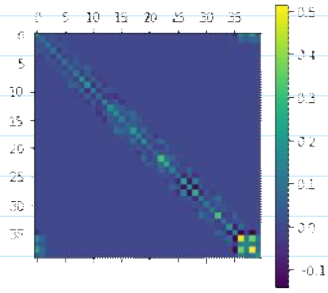




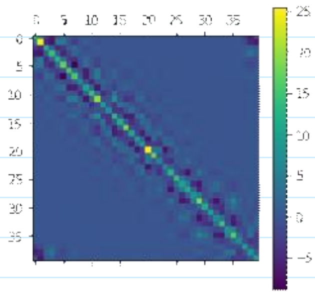
L



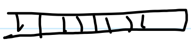
D^{-1}

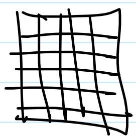


$L^T D^{-1} L \approx B^{-1}$



$[L^T D^{-1} L]^{-1} \approx \underline{B}$





NO

```
lr = LinearRegression(fit_intercept=False);
L = np.eye(n);
D = np.zeros((n,n));
D[0,0] = 1/np.var(DXk[0,:]); #We are estimating D^{-1}
for i in range(1,n):
    ind_neigh = np.arange(i-r,i+r+1)%n;
    ind_prede = ind_neigh[ind_neigh<i];
    y = DXk[i,:];
    X = DXk[ind_prede,:].T;
    lr_fit = lr.fit(X,y);
    err_i = y-lr_fit.predict(X);
    D[i,i] = 1/np.var(err_i);
    L[i,ind_prede] = -lr_fit.coef_;
```

$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D=

NEIGH OF i

CHOOSE PREDECESSOR

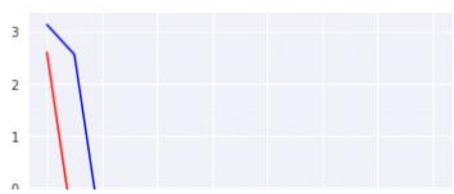
$y = X\beta + \varepsilon$

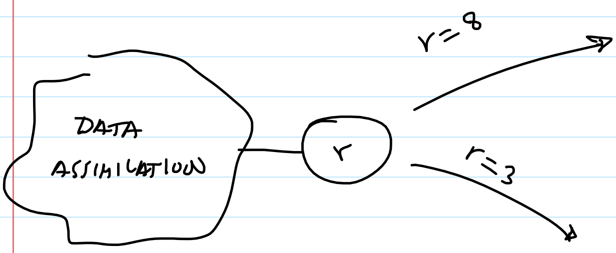
$\{L\}_{ij} = -\beta$

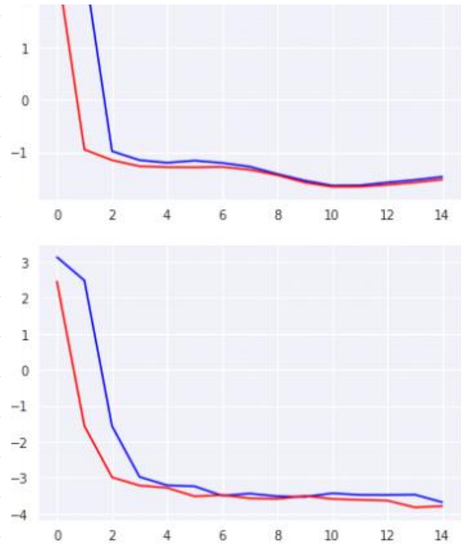
MODIFIED CHOLESKY DECOMPOSITION

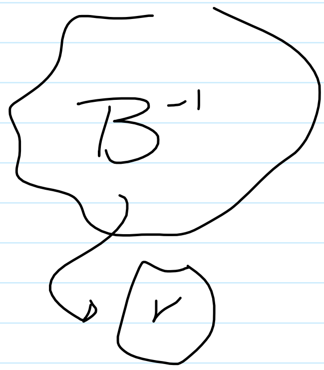
BOUNDARY CONDITIONS

Asimilación con Descomposición de Cholesky Modificada



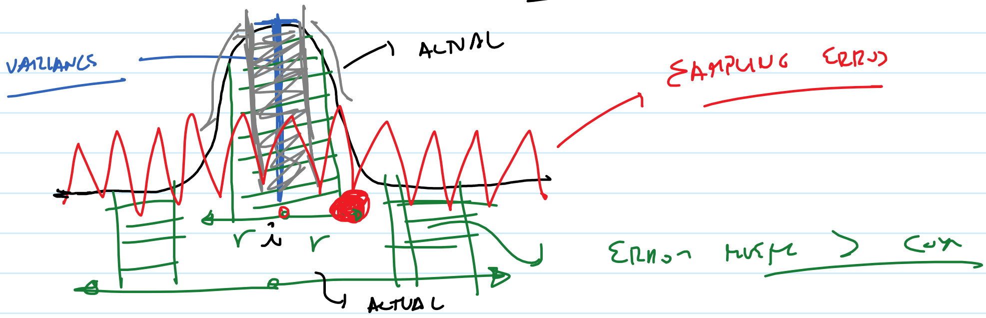






$$B^{-1}(r) = \underbrace{\underbrace{L(r)^T}_{L} \underbrace{D(r)^{-1}}_{L} \underbrace{L(r)}_{L}}_{L}$$

CAN BE SEEN A FUNCTION OF $\underline{\underline{r}}$



1

OR COVARIANCE

$$\longrightarrow \quad y = X\beta + \varepsilon$$

$$\beta^* = \arg\min_{\beta} \|y - X\beta\|^2 \Rightarrow \boxed{\beta^* = (\underbrace{X^T X}_{\text{COVARIANZA}})^{-1} X^T y}$$

$$X \in \mathbb{R}^{N \times p} \Rightarrow r \approx N$$

(a)

$$X = U \overset{\sigma_i}{\Sigma} V^T \Rightarrow \boxed{\beta^* = \sum_{i=1}^{p} \frac{1}{\sigma_i} v_i u_i^T y}$$

REAL

$$y = y^* + \tilde{y} \Rightarrow \beta^* = \sum_{i=1}^{p} \frac{1}{\sigma_i} \cdot v_i u_i^T \left[\underbrace{y^*}_{\text{ACTUAL}} + \overbrace{\tilde{y}}^{\text{RUIDO}}\right]$$

$$\Rightarrow \beta^* = \sum_{i=1}^{p} \frac{1}{\sigma_i} v_i \underbrace{U_i^T y^*}_{\phi_i^*} + \sum_{i=1}^{p} \frac{1}{\sigma_i} v_i \underbrace{U_i^T \tilde{y}}_{\tilde{\phi}_i}$$

VALIDA TRASH

$$\Rightarrow \beta^* = \sum_{i=1}^{p} \phi_i^* v_i + \sum_{i=1}^{p} \tilde{\phi}_i v_i$$

$$\boxed{\sigma_i \approx 0}$$

$$\phi_i^* = \frac{U_i^T y^*}{\sigma_i} \qquad \tilde{\phi}_i = \frac{U_i^T \tilde{y}}{\sigma_i} \Rightarrow$$

$U_i^T \tilde{y} > \sigma_i$ ✓

$U_i^T \tilde{y} < \sigma_i$ ✓

REGULARIZATION

$\|y - X\beta\|_2^2$ (NO REGULARIZATION) $\neq$

0.01

EVD

$\|y - X\beta^+\|_2^2$ $\quad \beta^+ = \sum_{i=1}^{z} \frac{U_i^T y}{\sigma_i} v_i$ $\quad \sigma_i >$ THRESHOLD (UMBRAL) ✓

1, EVD TRUNCATED

✓ $\|y - X\beta\|_2^2 + \lambda\|\beta\|_2^2$ → RIDGE / TIKHONOV] REGULARIZATION

✓ $\|y - X\beta\|_2^2 + \lambda\|\beta\|_1^2 \Rightarrow$ LASSO

✓ $\|y - X\beta\|_2^2 + \|\beta\|_H^2 \Rightarrow$ HUBER NORM → L1 — LASSO; L2 — RIDGE