① EnKF

$$X^a = X^b + P^b H^T (R + H P^b H^T)^{-1} D$$

$$X^a = X^b + P^b H^T Z \qquad (R + H P^b H^T)\, Z = D$$

↳ EnKF - Cholesky

$$x = \bar{x}^b + \Delta X w$$

$$[I + Q^T R^{-1} Q]\, W = Q^T R^{-1} D \quad ✓$$

$$\Rightarrow \quad X^a = X^b + \Delta X \cdot W$$

$d = y - H\bar{x}^b$ $\qquad$ $H\Delta X = Q \in \mathbb{R}^{m \times N}$

$$J(\bar{x}^b + \Delta X w) = \frac{1}{2}\|w\|^2 + \frac{1}{2}\| y - H(\bar{x}^b + \Delta X w)\|^2_{R^{-1}}$$

$y^{s(e)} - H x^{b(e)}$

$$\nabla_w J(\bar{x}^b + \Delta X w) = w - Q^T R^{-1}(d - Qw)$$

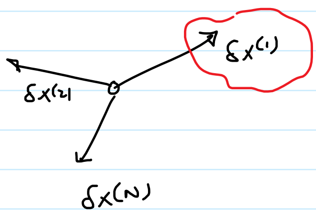
$$\Rightarrow w^* \Rightarrow (I + Q^T R^{-1} Q)\, w^* = Q^T R^{-1} d$$

↳ EnKF - SVD

$$X^a = X^b + \widehat{\Delta X}\, \hat{Q}^T Z \qquad \hat{Q} = H \widehat{\Delta X} \qquad \widehat{\Delta X} = \frac{1}{\sqrt{N-1}} \Delta X$$

$\Rightarrow$ $R + H P^b H^T$] covariance $\qquad \eta^{(e)} \sim \mathcal{N}(0, R + H P^b H^T)$

$$\Delta X = [\delta x^{(1)}, \delta x^{(2)}, \ldots, \delta x^{(N)}] \in \mathbb{R}^{n \times N}$$



$\delta x^{(e)} \sim \mathcal{N}(0, B)$

$\eta^{(1)}, \ldots, \eta^{(N)}$

↳ $H = U \Delta U^T$

$H^{1/2} = U \Delta^{1/2} U^T \approx U \Sigma V^T$

$\varepsilon_1 \sim \mathcal{N}(0, I)$

$\Rightarrow H = [U \Delta^{1/2} U^T][U \Delta^{1/2} U^T]$

$U \Delta^{1/2} I \Delta^{1/2} U^T$

$U \Sigma I \Sigma U^T$

$V^T V$

$R^{1/2} \varepsilon_1 \sim \mathcal{N}(0, R)$

$R^{1/2} \varepsilon_1 + B^{1/2} \varepsilon_2 \sim \mathcal{N}(0, R + B)$

$R^{1/2} \varepsilon_1 + H B^{1/2} \varepsilon_2 \sim \mathcal{N}(0, R + H B H^T)$

$\phi_{(j)} \sim \mathcal{N}(0, R)$ $\qquad$ $\phi_{(e)} \sim \mathcal{N}(0, HBH^T)$

$H\,\delta x^{(e)}$

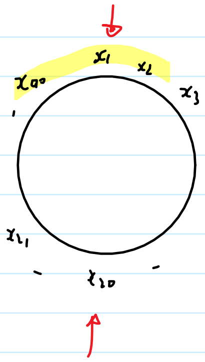
$$\mathbb{E}(\Phi\Phi^T) = R \qquad \mathbb{E}(\Delta x \Delta x^T) = B$$

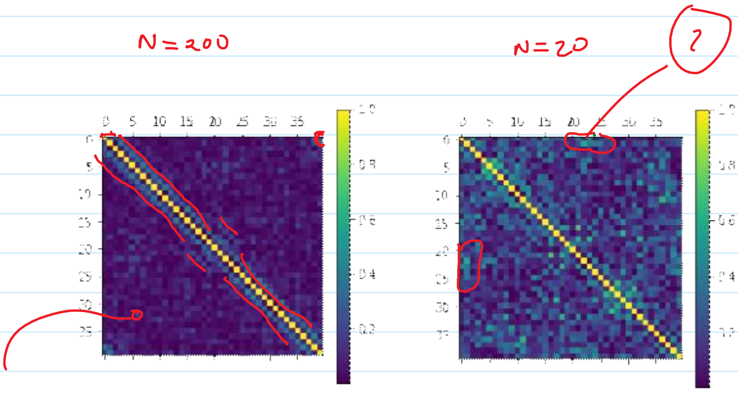
$$\Rightarrow \mathbb{E}(\Phi\Phi^T + H\Delta x \Delta x^T H^T) = R + HBH^T$$

$$\Rightarrow \mathbb{E}(\Phi + H\Delta x) = (R + HBH^T)^{1/2}$$

$U\Sigma V^T$

④ IMPLEMENTACIONS EFICIENTS





CORRELACIONS ESPÚRIES

$m=1$

⑤

$$X^a = X^b + P^b H^T (R + HP^bH^T)^{-1} D$$

$$\bar{x}^a = \bar{x}^b + P^b H^T (R + HP^bH^T)^{-1} d \qquad d = y - H\bar{x}^b$$

VECTOR DE INNOVACIÓ

$[1\ 0\ 0\ 0\ \cdot\ 0]$ $\quad H$

$$\bar{x}^a = \bar{x}^b + P^b H^T (\sigma_o^2 + \sigma_{k,k}^{b2})^{-1} d$$

$$\bar{x}^a = \bar{x}^b + P^b H^T \frac{d}{\sigma^2 + \sigma_{k,k}^{b2}}$$

K

?

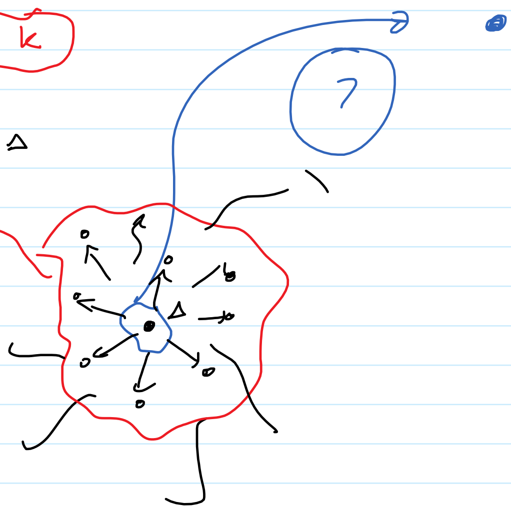
$$\bar{x}^a = \bar{x}^b + P^b H^T \frac{d}{\sigma^2 + \sigma_{k,k}^{b\,2}}$$

K → Δ

$$\bar{x}^a = \bar{x}^b + K\,\Delta$$

ok

INNOVACIÓN



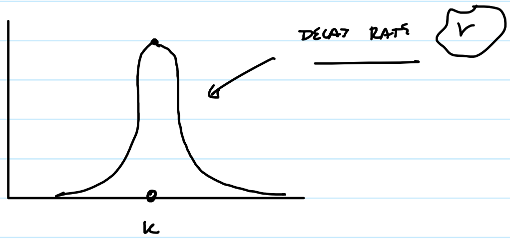
+ LOCALIZACIÓN

EVITA EL IMPACTO DE CORRELACIONES ESPURIAS

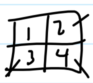
→ DIVERGENCIA DEL FILTRO

+ GAUSSIAN MODEL

+ RADIO DE INFLUENCIA

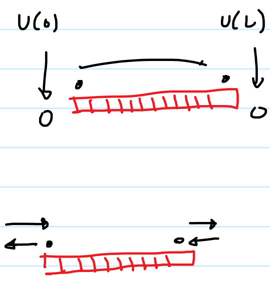


+ $f(k,\cdot) = e^{-\frac{1}{2}\frac{d(k,\cdot)^2}{r^2}}$ } DECORRELATION FUNCTION



LABEL

1 CONDICIONES DE FRONTERA



DIRICHLET HOMOGENEAS → BORDE?

PERIODIC PERIODICAS

+ ROW-MAJOR FORMAT

+ COLUMN-MAJOR FORMAT

RADIO DE INFLUENCIA

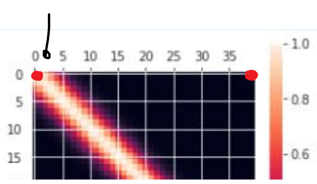
$$f(i,j,r) = \exp\left(-\frac{1}{2}\cdot\frac{d_{i,j}^2}{r^2}\right)$$

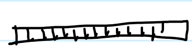
COMPONENTE i

COMPONENTE j

OK FOR LORENZ 96?

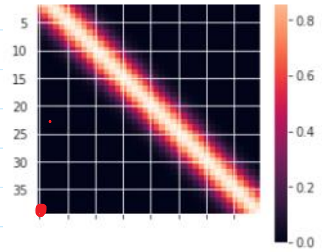
OK FOR HOMOGENEOUS





$|i-j|$ ✓

i ✓



39 40 1 2 3 → j

(1)

ok

39

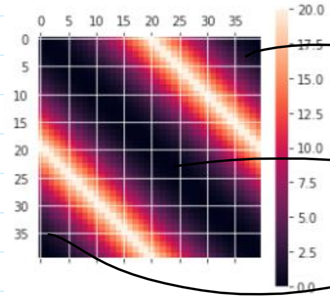
1 → 40 ⇒ (1)

i j

$(n-j)+i$?

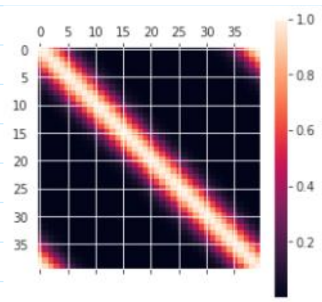
1 → 40

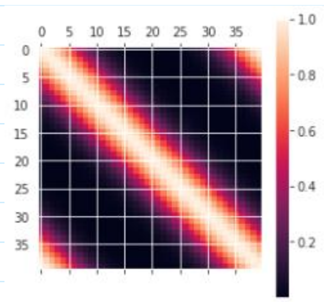
i j (1)

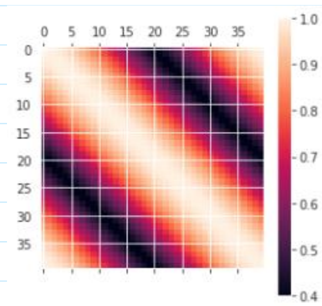


STRONG RELATIONS

r=3 r=5 r=14





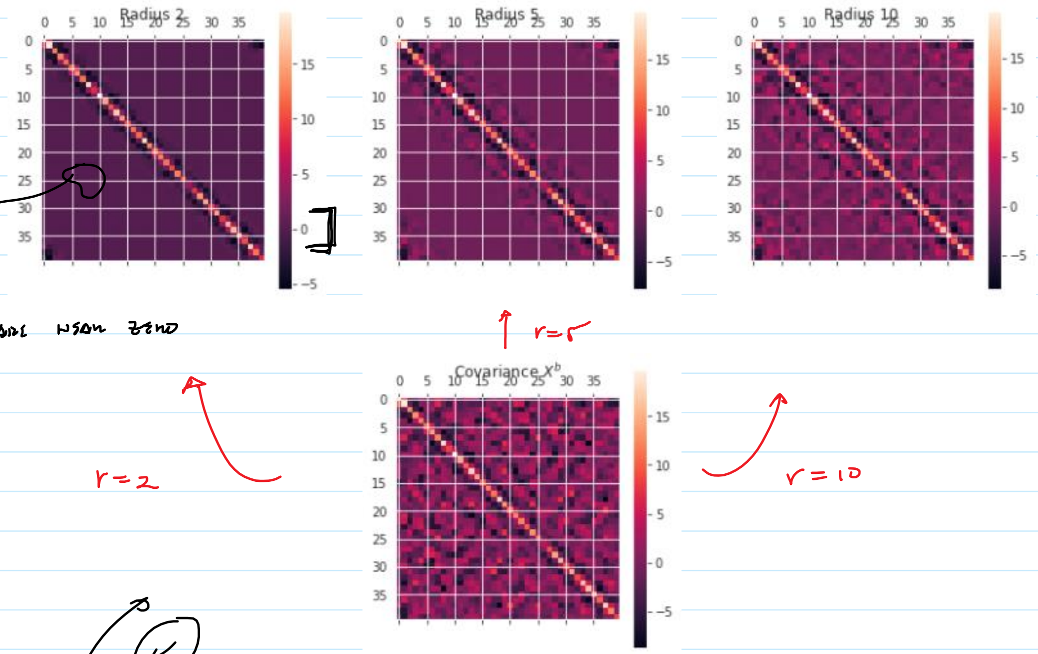


(*) SCHUR PRODUCT

P^b → COVARIANCE MATRIX FROM X^b

$$\hat{P}^b = L \circ P^b$$

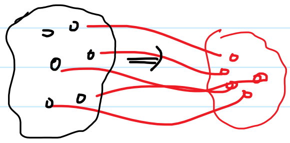
→ COMPONENT WISE MULTIPLICATION



CORRELATIONS ARE NON ZERO

(1) LOCALIZACIÓN

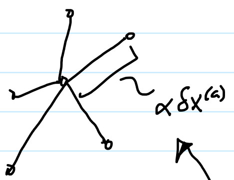
(2) INFLACIÓN



⇒ COLAPSE

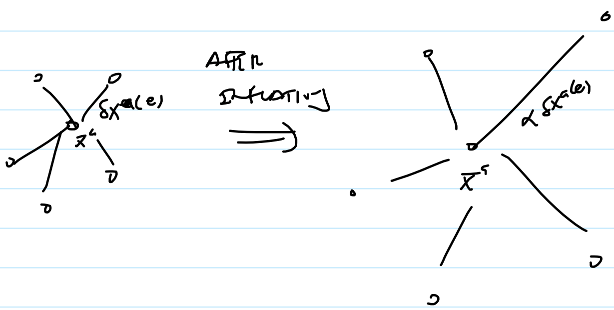
$\alpha = 1.02$

$= 1.05$ (DISTINTAS)



$\sim \alpha \, \delta X^{(a)}$

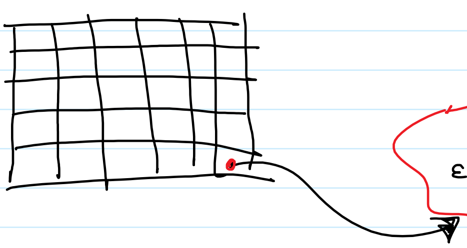
$$X^a = \bar{x}^a \cdot 1_N^T + \alpha \Delta X^a$$



OK

$c(\alpha)$

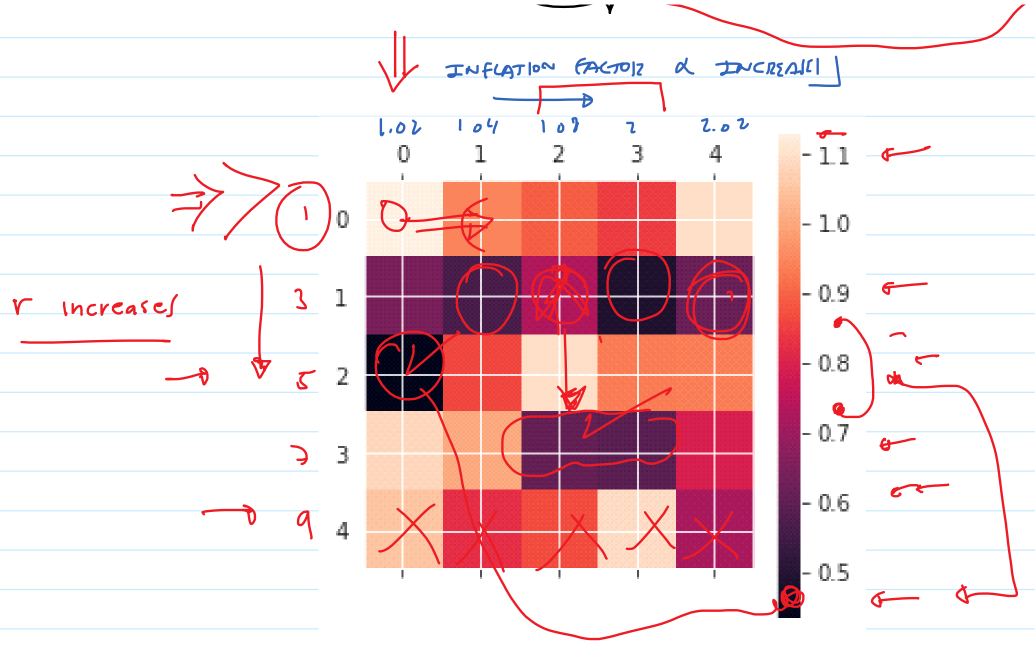
r



(r, α)

$$\varepsilon = \frac{1}{T} \sum_{i=1}^{T} \| x_i^a - x_i^* \|$$

⇓ INFLATION FACTOR α INCREASE



(4)

$$\nabla J(x) = B^{-1}(x - x^b) - H^T R^{-1}(y - Hx)$$

$$\nabla J_k = B^{-1}(x_k - x^b) - H^T R^{-1}(y - Hx_k) \quad (=) \; ?$$

$$x_{k+1} = x_k - \varepsilon \cdot \nabla J_k$$

LEARNING RATE

$y^{(e)} - H\, x^{b(e)}$

$$x^a - x^b = [B^{-1} + H^T R^{-1} H)^{-1} H^T R^{-1} d$$

$$x^a = x^b + [B^{-1} + H^T R^{-1} H]^{-1} H^T R^{-1} d$$

$$x^a = x^a_{2n} + \alpha \Delta x^a_k \quad ?$$

$$x^a = x^b + P^b H^T (R + H P^b H^T)^{-1} D$$

$$= x^b + \hat{\Delta x} \hat{Q}^T (R + H \hat{\Delta x} \hat{\Delta x}^T H^T)^{-1} D$$

$$= x^b + \Sigma V P$$

(7) INFLACIÓN OK!

$$= x^b + \text{Cholesky}$$

$P^b \to L \cdot P^b$

CAN BE FULL-RANK

$$= x^b + \Sigma M F$$

$$= x^b + \hat{P}^b H^T (R + H \hat{P}^b H^T)^{-1} D$$

$$x^a = x^b + \hat{P}^b H^T (R + H\hat{P}^b H^T)^{-1} D$$

[illegible]

t → BACKGROUND ⟹ MODEL ⟹ NUMERICAL INTEGRATION

t → ANALYSIS STEP ⟹ $x^a = x^b + P^b H^T z = x^b + \tilde{z}$

$$[B^{-1} + H^T R^{-1} H]\tilde{z} = H^T R^{-1} D$$

$n = 10^{\circ}$ / $\varphi 6$

$P^b = \hat{\Delta x}^b \hat{\Delta x}^{bT}$

EVD — Cholesky — EMF

RANK DEFICIENCY P^b

COVARIANCE LOCALIZATION

$\hat{P}^b = L \otimes P^b$

$x^a = x^b + \hat{P}^b H^T \hat{z}$

↓

$(R + H\hat{P}^b H^T)\hat{z} = D$

COVARIANCE INFLATION